hep-ph/0409281
February 2, 2008

On ϕ_3 Measurements Using $B^- \to D^* K^-$ Decays

Alex Bondar*[1] and Tim Gershon†[2]

[1]*Budker Institute of Nuclear Physics, Novosibirsk*
[2]*High Energy Accelerator Research Organization (KEK), Tsukuba*

Abstract

We point out that in decays of the type $B^- \to D^* K^-$, where the neutral D^* meson is an admixture of D^{*0} and $\bar{D}^{*0}$, there is an effective strong phase difference of π between the cases that the D^* is reconstructed as $D\pi^0$ and $D\gamma$. We consider the consequences for measurements of ϕ_3 using these modes, some of which are profound and beneficial.

PACS numbers: 11.30.Er, 12.15.Hh, 13.25.Hw, 14.40.Nd

* a.e.bondar@inp.nsk.su
† gershon@bmail.kek.jp

Measurement of the Unitarity Triangle angle ϕ_3 [1] is a major challenge. A number of methods have been proposed using $B^\mp \to DK^\mp$ decays, including those where the neutral D meson is reconstructed as a CP eigenstate (GLW) [3], in a suppressed final state (ADS) [4], or in a self-conjugate three-body final state, such as $K_S\pi^+\pi^-$ (Dalitz) [5]. Each of these approaches, while theoretically clean, has some difficulty for ϕ_3 extraction due to intrinsic ambiguities or model dependence; these can be overcome by combining the results from different techniques, and including other modes, such as $B^\mp \to D^*K^\mp$ and $B^\mp \to DK^{*\mp}$. The first studies of these modes, and constraints on ϕ_3, have recently begun to appear from the B factory experiments. In this note, we consider the effect of reconstructing the neutral D^* meson as either $D\pi^0$ or $D\gamma$, and examine the consequences for ϕ_3 measurements. It is found that the ADS method is significantly enhanced, and that ϕ_3 can be extracted by applying this technique to $B^\mp \to D^*K^\mp$ decays alone.

The basic idea, for all of these approaches, is that the neutral $D^{(*)}$ meson produced in $B^- \to D^{(*)}K^-$ is an admixture of $D^{(*)0}$ (produced by a $b \to c$ transition) and $\bar{D}^{(*)0}$ (produced by a colour-suppressed $b \to u$ transition) states. If the final state is chosen so that both $D^{(*)0}$ and $\bar{D}^{(*)0}$ contribute, the two amplitudes interfere. The resulting observables are sensitive to ϕ_3, the relative weak phase between the two B decay amplitudes.

In the general case of $B^\mp \to D^{(*)}K^\mp$, with $D^{(*)}$ decaying to a final state f, we can write the decay rates for B^- and B^+ ($\Gamma_\mp$), and the charge averaged rate ($\Gamma = (\Gamma_- + \Gamma_+)/2$) as

$$\Gamma_\mp \propto r_B^2 + r_D^2 + 2r_Br_D\cos\left(\delta_B + \delta_D \mp \phi_3\right), \tag{1}$$

$$\Gamma \propto r_B^2 + r_D^2 + 2r_Br_D\cos\left(\delta_B + \delta_D\right)\cos\left(\phi_3\right), \tag{2}$$

where the ratio of B decay amplitudes is usually defined to be less than one,

$$r_B = \frac{\left|A\left(B^- \to \bar{D}^{(*)0}K^-\right)\right|}{\left|A\left(B^- \to D^{(*)0}K^-\right)\right|}, \tag{3}$$

and the ratio of $D^{(*)}$ decay amplitudes is then defined as

$$r_D = \frac{\left|A\left(D^{(*)0} \to f\right)\right|}{\left|A\left(\bar{D}^{(*)0} \to f\right)\right|}. \tag{4}$$

The strong phase differences between the B and $D^{(*)}$ decay amplitudes are given by δ_B and δ_D, respectively. Note that r_B and δ_B take different values for different B decays; the values for $B^- \to DK^-$ and $B^- \to D^*K^-$ are not the same. On the other hand, the value of r_D depends only on the final state of the D decay, since the amplitudes for D^{*0} and $\bar{D}^{*0}$ decays to D^0 and $\bar{D}^0$, respectively, via emission of either a pion or a photon, will cancel in the ratio. For the GLW analysis, $r_D = 1$ and δ_D is trivial (either zero or π), while in the Dalitz analysis r_D and δ_D vary across the Dalitz plot, and depend on the D decay model used. For the ADS analysis, the values of r_D and δ_D are not trivial, and this is the main subject of our attention.

Let us consider the neutral D^* meson produced in B^- decay, which we denote by $\tilde{D}^*$:

$$\tilde{D}^* = D^{*0} + r_Be^{i(\delta_B - \phi_3)}\bar{D}^{*0}. \tag{5}$$

We define CP eigenstates of the neutral D^* system, following the phase convention $CP(D^{*0}) = \bar{D}^{*0}$, $CP(\bar{D}^{*0}) = D^{*0}$ [6], so that

$$D^*_+ = \frac{D^{*0} + \bar{D}^{*0}}{\sqrt{2}}, \qquad D^*_- = \frac{D^{*0} - \bar{D}^{*0}}{\sqrt{2}}, \tag{6}$$

and

$$D^{*0} = \frac{D^*_+ + D^*_-}{\sqrt{2}}, \qquad \bar{D}^{*0} = \frac{D^*_+ - D^*_-}{\sqrt{2}}, \tag{7}$$

and use similar definitions in the neutral D system. Thus

$$\tilde{D}^* = \frac{D^*_+ + D^*_-}{\sqrt{2}} + r_B e^{i(\delta_B - \phi_3)} \frac{D^*_+ - D^*_-}{\sqrt{2}}. \tag{8}$$

We now consider decays of the D^* CP eigenstates to $D\pi^0$ and $D\gamma$. Using η_X to denote the CP eigenvalue of X, we find that in the former case, $\eta_{D^*} = \eta_D \times \eta_{\pi^0} \times (-1)^l$, where the angular momentum between the D and π^0 mesons is required to take the value $l = 1$ by conservation of angular momentum. Thus $\eta_{D^*} = \eta_D$, and $D^*_\pm \to D_\pm \pi^0$. In the latter case, we have $\eta_{D^*} = \eta_D \times \eta_\gamma \times (-1)^l$; in this case we need to consider conservation of parity to find again that $l = 1$. Thus $\eta_{D^*} = -1 \times \eta_D$, and so $D^*_\pm \to D_\mp \gamma$.

Next we consider the neutral D meson produced in the decay $B^- \to \tilde{D}^* K^-$, $\tilde{D}^* \to \tilde{D}\pi^0$,

$$\begin{aligned} \tilde{D} &= \frac{D_+ + D_-}{\sqrt{2}} + r_B e^{i(\delta_B - \phi_3)} \frac{D_+ - D_-}{\sqrt{2}} && (9) \\ &= D^0 + r_B e^{i(\delta_B - \phi_3)} \bar{D}^0, && (10) \end{aligned}$$

whereas that produced in the decay $\tilde{D}^* \to \tilde{D}\gamma$ is given by

$$\begin{aligned} \tilde{D} &= \frac{D_- + D_+}{\sqrt{2}} + r_B e^{i(\delta_B - \phi_3)} \frac{D_- - D_+}{\sqrt{2}} && (11) \\ &= D^0 - r_B e^{i(\delta_B - \phi_3)} \bar{D}^0 && (12) \\ &= D^0 + r_B e^{i(\delta_B + \pi - \phi_3)} \bar{D}^0. && (13) \end{aligned}$$

Hence there is an effective strong phase shift of π between the two cases [7].

One should also consider the effect of other nontrivial strong phases which may be produced in D^* decay. Although these may exist, we are interested only in the strong phase difference between $B^- \to D^{*0} K^-$ and $B^- \to \bar{D}^{*0} K^-$ amplitudes; any such additional phase will be the same for both D^{*0} and $\bar{D}^{*0}$ decays, and thus cancel when the difference is considered, due to CP invariance of the D^* decay amplitudes.

We now consider how this realization affects the various approaches used for ϕ_3 extraction. In the GLW technique, the D^* meson is reconstructed in CP eigenstates. If the CP eigenvalues are properly taken into account, there is no advantage, except for statistical gain, in using both $D^* \to D\pi^0$ and $D^* \to D\gamma$ decays. However, experimentalists should take care to account for cross-feed between these modes, which can pollute the CP content.

In the Dalitz analysis technique, the strong phase shift of π appears between the two D^* decay modes, and this should be taken into account in the analysis. This method also allows a straightforward cross-check of our conclusion; the strong phase can be measured from independent fits to samples with $D^* \to D\pi^0$ and $D^* \to D\gamma$, and the two cases should be found to differ by the factor of π. The BaBar collaboration has recently released preliminary results of such an analysis in which both D^* decay modes are utilized [8]. The treatment of the strong phases is not entirely clear; in case the strong phase shift of π is not taken into account the most likely effect is a reduction in the sensitivity to ϕ_3.

It is for the ADS technique that the impact is most significant. In this approach, it is convenient to measure the ratios of rates for B decays to suppressed and favoured final

states, $\mathcal{R}$ so that the constant of proportionality drops out of Eq. 1. If we consider such ratios separately for B^- and B^+, for both cases $D^* \to D\pi^0$ and $D^* \to D\gamma$, we see

$$\mathcal{R}_{\mp}\left(D^* \to D\pi^0\right) = r_B^2 + r_D^2 + 2r_B r_D \cos\left(\delta_B + \delta_D \mp \phi_3\right), \tag{14}$$

$$\mathcal{R}_{\mp}\left(D^* \to D\gamma\right) = r_B^2 + r_D^2 - 2r_B r_D \cos\left(\delta_B + \delta_D \mp \phi_3\right). \tag{15}$$

Note that here, in contrast to Eq. 1, the strong phase difference of the D^* decay is explicitly taken into account so that δ_D is the strong phase difference of the D decay amplitudes.

Since there are four independent equations, which contain only three unknowns (the value of r_D is known from D decay), ϕ_3 can be extracted, up to the usual four-fold ambiguity. Compare this to the standard ADS analysis; for any particular B decay one finds only two independent equations, which cannot be solved for three unknowns. The usual resolution [9] is to combine decay modes, such as $B^\mp \to DK^\mp$ and $B^\mp \to D^*K^\mp$, but for each new mode two extra unknowns (r_B, δ_B) are added.

The charge averaged rates are given by

$$\mathcal{R}\left(D^* \to D\pi^0\right) = r_B^2 + r_D^2 + 2r_B r_D \cos\left(\delta_B + \delta_D\right)\cos\left(\phi_3\right), \tag{16}$$

$$\mathcal{R}\left(D^* \to D\gamma\right) = r_B^2 + r_D^2 - 2r_B r_D \cos\left(\delta_B + \delta_D\right)\cos\left(\phi_3\right), \tag{17}$$

so that

$$\left(\mathcal{R}\left(D^* \to D\pi^0\right) + \mathcal{R}\left(D^* \to D\gamma\right)\right)/2 = r_B^2 + r_D^2, \tag{18}$$

does not depend on any phases. Hence the value of r_B in $B^- \to D^*K^-$ decays can be straightforwardly obtained.

Recently, the BaBar collaboration has performed an ADS analysis using $B^\mp \to D^*K^\mp$, using the subsequent doubly Cabibbo-suppressed decay $D \to K\pi$ [10]. In the absence of any significant signals, they combine the likelihoods for $\mathcal{R}\left(D^* \to D\pi^0\right)$ and $\mathcal{R}\left(D^* \to D\gamma\right)$, and obtain an upper limit on the quantity on the left-hand side of Eq. 18 of 0.021 at 90% confidence level (C.L.). This is then converted into an upper limit of $r_B < 0.21$ (90% C.L.). (For the suppressed $K\pi$ final state, the ratio of D decay amplitudes is known to be $r_D = 0.060 \pm 0.003$ [2].) Since the strong phase shift of π is not taken into account, this limit is calculated using Eq. 16, and allowing for any possible values of $\delta_B + \delta_D$ and ϕ_3. However, if the upper limit is calculated using Eq. 18, a more stringent value of $r_B < 0.134$ can be obtained.

In conclusion, we have shown that in decays of the type $B^- \to D^*K^-$, there is an effective strong phase difference of π between the cases that the D^* is reconstructed as $D\pi^0$ and $D\gamma$. We have examined the consequences for several approaches to extract ϕ_3, and find a significant benefit for the ADS technique. In future this enhanced ADS method may be used to measure ϕ_3 using $B^\mp \to D^*K^\mp$ decays alone.

We are grateful to V. Chernyak, A. Vainshtein, M. Voloshin and H. Yamamoto for useful discussions. T. G. is supported by the Japan Society for the Promotion of Science. This work was carried out during a visit to Japan by A. B., supported by KEK.

[1] For a review of CP violation phenomenology, see D. Kirkby and Y. Nir in [2].
[2] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).

[3] M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991), M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991).
[4] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997), Phys. Rev. D **63**, 036005 (2001).
[5] A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D **68**, 054018 (2003); A. Poluektov *et al.* (Belle Collaboration), `hep-ex/0406067` (to appear in Phys. Rev. D).
[6] The choice of CP phase convention does not affect observable quantities.
[7] This conclusion can also be obtained invoking only C symmetry in D^* decays.
[8] B. Aubert *et al.* (BaBar Collaboration), BaBar-CONF-04/043, `hep-ex/0408088`.
[9] D. Atwood and A. Soni, `hep-ph/0312100`.
[10] B. Aubert *et al.* (BaBar Collaboration), BaBar-CONF-04/013, `hep-ex/0408028`.